Behavior of Wall Jet in Laminar-to-Turbulent Transition

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Abstract

RESULTS are presented concerning the linear stability analysis of an incompressible plane wall jet flow, i.e., flow tangent to a surface and interacting with a still fluid. The numerical solutions of the Orr-Sommerfeld equation are obtained by means of a finite-element approximation. Two different unstable modes are found and two neutral curves are plotted, which show an overlapping region of instability for both types of modes. Experimental flow is realized, and a visualization is compared with a computer-simulated flowfield.

Contents

The stability of the wall jet and the transition of this flow from laminar to turbulent are of interest for different applications.

The instability phenomena of the laminar plane wall jet were studied by several authors.[1-5] The temporal linear analysis[1,4] gives a second neutral curve within the first; in Ref. 1, this curve is attributed to a higher mode, whereas in Ref. 4 the authors concluded that the inside of the second curve was a stable region.

In the present work it is shown that two different modes lead to instability of this flow. A picture of the well-organized structures developing at the onset of transition is also obtained by numerical simulation, and compared with flow visualization.

Reducing the Navier-Stokes equation to the Orr-Sommerfeld equation gives

$$L\varphi = 0 \tag{1}$$

With the fourth-order linear differential operator

$$L = (D^2 + \alpha^2)^2 + i\alpha Re\left[(U(y) - c)(D^2 + \alpha^2) + \frac{d^2 U}{dy^2} \right] \tag{2}$$

where

$$D = -i \frac{d}{dy} \tag{3}$$

Re is the Reynolds number, φ, α, and c define the Fourier components of the disturbance

$$u' = \frac{d\varphi}{dy} \exp[i\alpha(x - ct)] \tag{4}$$

$$v' = -i\alpha\varphi \exp[i(x - ct)] \tag{5}$$

where φ is a complex function of the real variable y; the wave number α is real and the frequency $\omega = \alpha c$ is a complex quantity, c being the complex eigenvalue (c_R is the phase velocity and c_I the amplification rate).

The numerical method applied here was used in Refs. 6 and 7 in bounded and unbounded domains. For the wall jet, the boundary conditions are:

$$\varphi = D\varphi = 0 \text{ for } y = 0 \quad \text{and} \quad \varphi, D\varphi \to 0 \text{ for } y \to \infty \tag{6}$$

In a finite-element scheme, Hermitian cubic polynomials are used as trial/test functions in the interval $[0, y_\infty]$, while a special "bubble" function is used over the infinite element $[y_\infty, +\infty]$. The shape function chosen is related to the inviscid solutions of the asymptotic behavior of φ for $y > y_\infty$.[6,7] The basic velocity profile adopted for calculation is obtained by interpolating Glauert's laminar profile[8] by Hermitian cubic polynomials. The velocity is considered constant far enough from the wall

$$U(y) = U_\infty = 0 \qquad \forall y \in [y_\infty, +\infty] \tag{7}$$

The velocities are normalized by the maximum velocity $U_m \propto x^{-1/2}$; the lengths by $\delta \propto x^{3/4}$, corresponding to $U = U_m/2$.

For computing, we have taken 25 internal mesh points and have assumed that $y_\infty = 4\delta$.

The two neutral stability curves obtained (Fig. 1) agree with the curves plotted in Ref. 4. In zone I of the α-Re plane only one eigenvalue has an imaginary part $c_I > 0$ (unstable mode), whereas no unstable modes appear within zone III.

The eigenfunctions corresponding to the "unstable" eigenvalues show two different shapes. In zone Ia the unstable mode yields a u' disturbance whose modulus reaches a max-

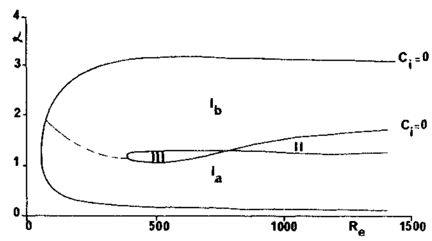

Fig. 1 Neutral stability curves in α-Re plane.

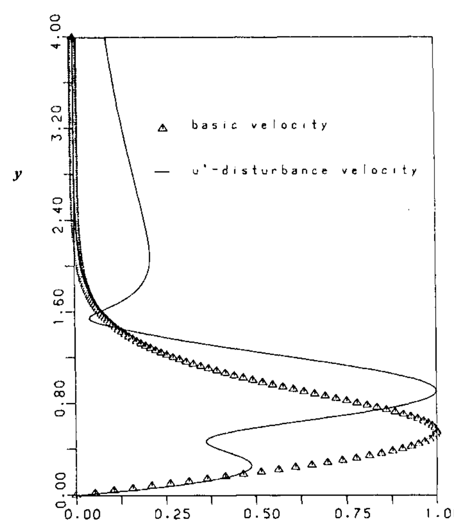

Fig. 2 Basic velocity and u'-disturbance velocity corresponding to the conditions $Re = 200$ and $\alpha = 0.5$.

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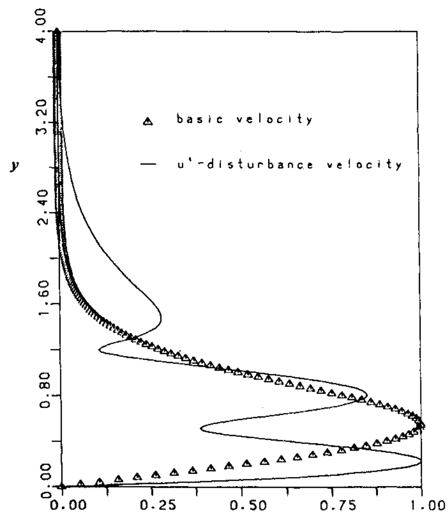

Fig. 3 Basic velocity and u'-disturbance velocity corresponding to the conditions $Re = 200$ and $\alpha = 2.0$.

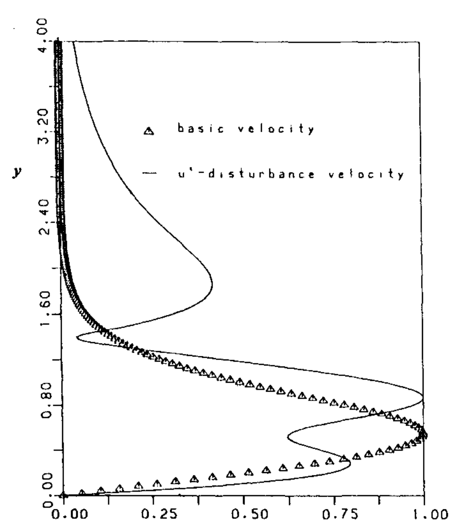

Fig. 4 Basic velocity and u'-disturbance velocity corresponding to the critical conditions $Re_c = 56.68$ and $\alpha_c = 1.16$.

imum in the outer region, in zone Ib the modulus of the u' disturbance has a maximum within the inner region close to the wall. Figures 2 and 3 show this modulus for $\alpha = 0.5$, $Re = 200$ and $\alpha = 2.0$, $Re = 200$, respectively. In zone II both kinds of modes are present. Figure 4 shows the modulus of the u' disturbance corresponding to the critical point $\alpha_c = 1.16$ and $Re_c = 56.68$, the maximum occurs at the inflection-point level in the outer region.

As conjectured in Refs. 2 and 3, the instability of a wall jet is caused both by the inflection point of the basic velocity profile and by viscous effects in the inner wall region, although the outer region disturbance leads to the critical Reynolds number.

The streakline pattern (the locus of particles passing through a fixed point of the flow at different times) was calculated at the critical conditions (α_c,Re_c) for the disturbed zone. The calculations were compared with an experiment.[9] Experimentally, the streaklines were visualized by injecting smoke into the flow (Fig. 5).

To plot a streakline for a fixed time t, one has to compute pathlines for various initial times t_0. The pathline equations are

$$\frac{dx}{dt} = u[x(t), y(t), t], \quad \frac{dy}{dt} = v[x(t), y(t), t] \tag{8}$$

The velocity components of the disturbed wall jet are

$$u(x,y,t) = U(y) + \frac{1}{2}\left\{\frac{d\varphi}{dy}\exp[i\alpha(x-ct)] + \text{c.c.}\right\} \tag{9}$$

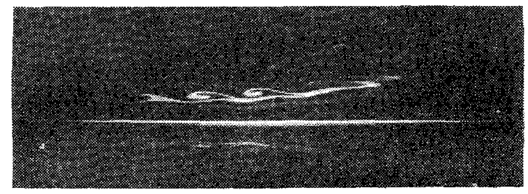

Fig. 5 Flow visualization of the wall jet during transition at low Reynolds number (exit velocity, 0.07 m/s).

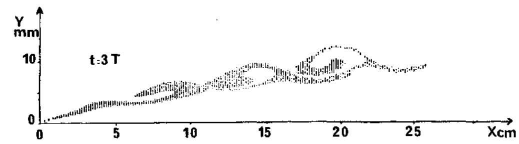

Fig. 6 Computer-simulated streaklines in dimensional form. Basic flow plus critical disturbance ($u'_{max}/U_m = 0.05$).

$$v(x,y,t) = \frac{1}{2}\{-i\alpha\varphi\exp[i\alpha(x-ct)] + \text{c.c.}\} \tag{10}$$

where c.c. denotes complex conjugate. Equations (8) were integrated by a Runge-Kutta method; a strip of ten markers with the center at $y = 0.93$ was injected at each step of integration. Figure 6 shows the streaklines at time $t = 3T$, where $T = 2\pi/\omega_c$ is the period critical disturbance.

Conclusion

The present numerical analysis of the stability of a plane wall jet confirms the existence of a second neutral curve bounding a stable region for $Re > Re_c$; this inner stable region is sharply reduced in the frequency domain.

Two kinds of unstable eigenmodes are present, which produce a particular transition to instability. In fact, the unstable large-scale disturbances have the highest values in the outer inflection point, while the unstable small-scale disturbances have the highest values close to wall. Hence, the outer inflection point renders the laminar jet vulnerable to the large-scale oscillations, while the effects of the viscous instability seem to yield the amplification of the small-scale oscillations.

Numerically computed flowfields obtained by superimposing the critical disturbance on the basic velocity show a pattern of the transitional process. The streaklines obtained could correspond well to the coherent structures of large-scale eddies at the onset of turbulence.

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